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Bayesian Estimation of Directional Wave-Spectrum Using Vessel Motions and Wave-Probes: Proposal and Preliminary Experimental Validation

The measurement of the directional wave spectrum in oceans has been done by different approaches, mainly wave-buoys, satellite imagery and radar technologies; these methods, however, present some inherent drawbacks, e.g., difficult maintenance, low resolution around areas of interest and high cost. In order to overcome those problems, recent works proposed a motion-based estimation procedure using the vessel as a wave sensor; nevertheless, this strategy suffers from low-estimation capabilities of the spectral energy coming from periods lower than the cutoff period of the systems, which are important for the drift effect predictions. This work studies the usage of wave-probes installed on the hull of a moored vessel to enhance the estimation capabilities of the motion-based strategy, using a high-order estimation method based on Bayesian statistics. First, the measurements from the wave-probes are incorporated to the dynamic system of the vessel as new degrees-of-freedom (DOF); thus, the Bayesian method can be expanded without additional reasoning. Second, the proposal is validated by experiments conducted in a wave-basin with a scale model, concluding that the approach is able to improve not only the estimation of spectra with low peak period but also the estimation in the entire range of expected spectra. Finally, some drawbacks are discussed, as the effect of the nonlinear roll motion, which must be taken in account when calculating the wave-probe response; and the poor mean-direction estimation capability in some particular wave directions and low peak periods. [DOI: 10.1115/1.4039263]

Keywords: directional wave spectrum, Bayesian estimation, wave-probe

Introduction

The economical exploration of seas, mainly oil and natural gas in offshore fields, has motivated advanced studies and research about oceanic systems and environmental forces acting on them.

In order to reduce operating costs, those studies are conducted both aiming at the improvement of the systems design—acquiring statistical bases for scenario modeling, validating mathematical theories, and validating experimental model tests in towing tanks; and aiming at the amplification of safe operating windows—using real-time monitoring systems capable of predicting dynamic behaviors. In all of them, the correct modeling of the environmental forces plays a major role.

Oceanic systems—mostly moored offshore floating facilities—experience forces from wind, current and waves, and a number of models has been proposed to describe them properly, with different levels of complexity; however, the in loco measurement of those environmental actors has always been a challenge.

The waves, in particular, have been measured using meteorological buoys, satellite imagery, and radar reconstruction, but all of those solutions have problems, respectively: difficult maintenance and high rate of damage after extreme conditions; poor resolution around interesting regions; and high sensibility to installation errors and high maintenance and acquisition costs.

Recent works in the wave estimation area, however, were able to solve the problems presented using the oceanic system itself as a measurement buoy, what is called wave-buoy analogy. In this way, motion sensors already installed on the facilities, i.e., inertial units, can be used, reducing installation and maintenance costs; guaranteeing the maximum resolution in the area of interest—the vessel itself; and avoiding damage while monitoring extreme events.

Despite the improvements, the dynamical behavior of the systems presents a high cutoff period, around eight seconds, making them unable to predict an important range of the sea wave spectrum coming from periods lower than this threshold, which are important for second order effects like drift forces. Therefore, the methods are still not completely satisfactory.

The correct prediction of the drift forces is important for the mooring design of anchored systems, since they can induce both a constant force acting on the system, mean drift load, and a low-frequency harmonic component capable of inducing resonance, slow drift load. Furthermore, a real-time prediction of those forces can anticipate motions during high-risk operations.

Other systems that can benefit from better estimations are dynamic positioning (DP) systems, which use global positioning

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Fig. 1 Example of a vessel with wave-probe on the hull. Adapted from Ref. [1].

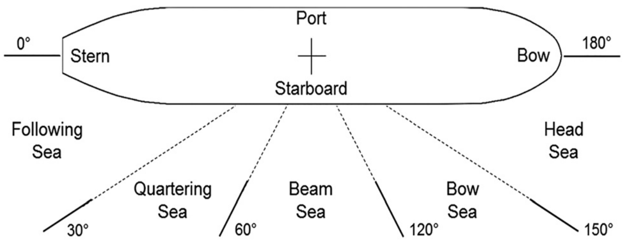

Fig. 2 Frame of reference for the incoming waves. A wave coming from 0 deg encounters the stern first and is called a following sea.

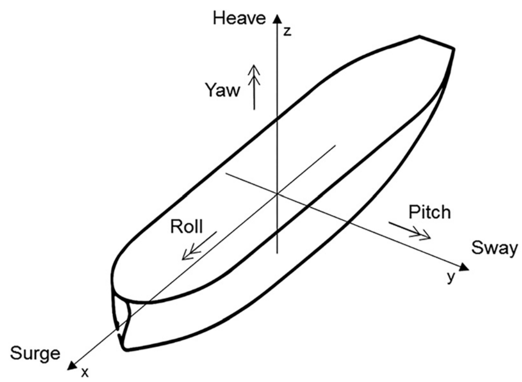

Fig. 3 Frame of reference for the vessel. A positive pitch causes a negative vertical translation of the bow of the vessel.

systems and thrusters commanded by a control law to guarantee a stationary vessel position; mainly in deep seas in which conventional mooring is not possible. It is known that a correct drift force prediction, used as a feed-forward compensator, is able to improve the DP control response, reducing the demanded power and increasing the safe operating window.

Focusing on those applications, this work proposes a possible solution for the natural constraints of the wave-buoy analogy, using wave-probes—gauges that measure the distance between themselves and the water surface—installed on the hull of the oceanic system as a complementary measuring device, Fig. 1.

The approach is justified by the behavior of low period waves encountering a free floating system, in which the system starts to behave like a wall, not moving with the wave and reflecting almost all the energy encountering it, i.e., amplifying the wave-elevation measurement from the wave-probes, regardless of the lack of motion. Hence, the proposal is able to incorporate low periods in the spectra estimation without incurring the problems presented so far.

Frames of Reference

For the sake of clarity, the following frames of reference will be used during this text, Figs. 2 and 3.

Literature Review

Research about experimental wave measurement has been conducted since the end of the second world war [2], but the usage of the high-order methods—methods capable of predicting each value in a discretized directional wave-spectrum—using the vessel motions, is a much more recent subject.

Particularly, the Bayesian Estimation of the directional wave-spectrum using the wave-buoy analogy was first addressed in 2000 by Iseki and Kohei [3]. In this preliminary work, most of the effort was being made to solve the so-called triple-valued function, which corrects the Doppler Effect that changes the frequency of a wave encountering a vessel with non-negligible forward speed.

In the following years, the studies were focused in the comparison of the Bayesian approach against other estimation methods, mainly parametric ones, which depends on the fitting of the measured motion responses by simulated motion responses from theoretical descriptions of the seas.

In 2003, in the context of dynamic positioned vessels, Tannuri et al. [4] set aside the triple-valued problem, as DP vessels are supposed to be stationary, and performed the first comparison between the Bayesian and the parametric estimation algorithms, giving a slightly preference to parametric models, after numerical and towing tank experimental tests.

In the work, the authors also proposed a change in the motion base used to estimate the sea, using the sway instead of the roll motion, usually adopted due to the wave-buoy analogy, arguing that the roll motion is affected by viscous nonlinear effects that would probably degrade its response calibration, while the sway motion has the same asymmetric behavior concerning the port and starboard incoming wave directions, which is indispensable for a good direction estimation.

Nielsen started his investigation into the subject in 2005, in his Ph.D. thesis [5]. In the work, he emphasized the limitation of motion-based methods because of the natural filtering properties of the dynamic system and compared the different estimation methods, although not giving preference to any. Nielsen would give a possible solution to the filtering problem some years later, [6], in the context of parametric estimation, validating numerically the usage of relative motions between the water surface and the vessel, which gives support to the idea of using wave-probes as a complementary measurement strategy.

Concerning parametric estimation, [7] put into perspective the nonlinearity and nonconvexity of the optimization problems involved in the estimation, suggesting the use of global search procedures as the genetic algorithm to find the best solution. The problem was also discussed by Nielsen [8], who showed that an improper first guest for gradient-based algorithms could potentially reach extremely bad solutions when using the parametric approach.

In 2007, a full-scale experimental campaign was described by Simos, following the previous work of his research group with Tannuri and co-workers [9]. In the work, the motion-based estimation using an floating production storage and offloading was compared against a wave-buoy measurement close to the location of the vessel. In this case, however, the data suggested that the Bayesian Estimation was more robust than the parametric model, demanding, at the same time, much less computational power.

In the following years, other comparisons and improvements in the Bayesian method were made, for example, Refs. [10–18]. In general, the improvements proposed in those papers are all included in the estimation presented in this work, and the comparisons are usually favorable to the Bayesian estimation against the parametric model, although Nielsen works do not suggest a clear preference. The results were also compared to estimations from wave-buoys, meteorological satellites, and radar imagery, with good consistence.

In conclusion, the Bayesian estimation was found to be a reliable algorithm, performing equally well or better than other estimation methods.

From the perspective of the DP systems, there are also papers that discussed wave-estimation, but they usually adopt methods that estimate the drift-forces in a more straightforward manner.

Most of the literature indicates Pinkster as the pioneer in the usage of wave estimation in feedforward compensators, [19]. In his work, Pinkster suggested a method to estimate the slow drift force by the direct integration of the relative motion measured in the hull of the vessel. He justified the feasibility of the method arguing that only a finite number of weighted sensors need to be used, as the water height varies slowly along the water-line. Despite that, a significant number of eight wave-probes were used, and only some forces could be estimated.

A more recent work from Aalbers, in which Pinkster is coauthor, [20], proposed a second method, using two wave-probes located symmetrically at the port and starboard, which are used to provide information about the direction of the incoming wave, and one wave-probe located at the bow of the ship, to acquire energy information. The second method is interesting for this work because it uses the drift force transfer functions to estimate the forces, justifying the high-order directional spectrum approach. Despite that, Aalbers concluded that the second method is generally worse than the Pinkster method and only works for head seas, although he recognizes the need of further developments.

Finally, a modern treatment of the wave feedforward usage in DP systems can be found in Refs. [21] and [22]. Initially, they explain the reason why first order forces cannot be counteracted by the traditional DP systems, since they are forces that change direction faster than the usual propellers are able to; despite this, they result in zero-mean motions. Second-order forces, however, which are described by high-order terms in the Bernoulli's equation, have nonzero mean, which eventually results in a nonnegligible motion, with a low-frequency component that can easily be close to the DP system resonance; consequently, they must be taken in account, at least in the design phase. The works also use the Pinkster approach, with twenty wave-probes around the hull, showing how the method is still the standard practice in this subject.

The proposal of this paper is to combine the two approaches, high-order Bayesian estimation using vessel motions and the Pinkster method, aiming at improving both: increasing the estimation range of the Bayesian algorithms and, at the same time, demanding less wave-probes than the Pinkster drift forces prediction method. The proposal can also be seen as an expansion of the idea of using relative motions discussed originally by Nielsen, evaluating, experimentally, high-order nonparametric estimation procedures for predicting the entire directional spectrum.

The Bayesian Estimation Algorithm

The Bayesian statistical allows the estimation of the probability of happening of the state $\mathbf{x}$, $P(\mathbf{x})$, after the performed measurement $\mathbf{y}$, Eq. (1), with $P(\mathbf{x}|\mathbf{y})$ the probability of $\mathbf{x}$ after the occurrence of $\mathbf{y}$

$$P(\mathbf{x}|\mathbf{y}) = P(\mathbf{y}|\mathbf{x}) \cdot P(\mathbf{x})/P(\mathbf{y}) \tag{1}$$

The rationale in the Bayesian estimation algorithms is, then, to choose the most probable state $\mathbf{x}$ as the response of the method. Ignoring $P(\mathbf{y})$, since the measurement was already taken and this term does not depend on $\mathbf{x}$, i.e., it is just a constant, the final algorithm can be stated as the maximization, with respect to the possible states $\mathbf{x}$, given in below equation

$$\max_{\mathbf{x}} P(\mathbf{y}|\mathbf{x}) \cdot P(\mathbf{x}) \tag{2}$$

The first term, $P(\mathbf{y}|\mathbf{x})$, is called likelihood function, and usually is related with the measurement noise. The second term, $P(\mathbf{x})$, is called prior function, and is related with the probability of each state $\mathbf{x}$. Since the last term is usually not known, the usual approach is to use a function representing a subjective belief about the state, as low energy or smoothness.

In the particular case of linear systems, $\mathbf{A}\mathbf{x} = \mathbf{b}$, with all error measurements independent Gaussian noises with equal magnitude, and prior function also a Gaussian distribution with general covariance $\mathbf{H}^{-1}$, the estimation can be represented by the minimization, with respect to the possible states $\mathbf{x}$, given in Eq. (3), with u a constant called hyperparameter, weighting the influence of the measurements versus the belief about the state. A straightforward demonstration of the equivalence between Eqs. (2) and (3), for Gaussian modeling of the likelihood and prior functions, is given in Ref. [12]

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2 + u^2 \mathbf{x}^{\mathrm{T}} \mathbf{H} \mathbf{x} \tag{3}$$

Assuming linearity between waves and vessel response, an integral formulation can be stated, Eq. (4), relating: the directional wave-spectrum $S(\omega, \beta)$, i.e., the energy contained in the waves coming from a particular direction β with a particular angular frequency ω; and the cross-spectra $\phi_{ij}(\omega)$ between the vessel degrees-of-freedom (DOF), two by two, in a particular frequency ω. The linear terms relating both are called response amplitude operators (RAOs) and are equal to the response of a particular DOF i or j, due to a wave coming from direction β, with angular frequency ω

$$\phi_{ij}(\omega) = \int_{-\pi}^{\pi} RAO_i(\omega, \beta) \cdot \overline{RAO}_j(\omega, \beta) \cdot S(\omega, \beta) \cdot d\beta \tag{4}$$

Discretizing the previous equation in K directions and M angular frequencies, assuming n DOFs, and separating the real and imaginary parts, it is possible to describe the integral formulation as a simple linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$, suitable for the Bayesian analysis, Eqs. (5)–(9)

$$\phi_{ij}(\omega_m) = \sum_{k=1}^{K} RAO_{\mathrm{i}}(\omega_m, \beta_k) \cdot \overline{RAO}_j(\omega_m, \beta_k) \cdot S(\omega_m, \beta_k) \cdot \Delta\beta \tag{5}$$

$$\mathbf{b}(\omega_m) = \begin{bmatrix} \boldsymbol{\varphi}_{ii}(\omega_m) \\ \mathrm{Re}\{\boldsymbol{\varphi}_{ij}(\omega_m)\} \\ \mathrm{Im}\{\boldsymbol{\varphi}_{ij}(\omega_m)\} \end{bmatrix}, \quad \boldsymbol{\varphi}_{ii} = \begin{bmatrix} \phi_{11} \\ \vdots \\ \phi_{nn} \end{bmatrix}, \quad \boldsymbol{\varphi}_{ij} = \begin{bmatrix} \phi_{12} \\ \vdots \\ \phi_{n-1n} \end{bmatrix} \tag{6}$$

$$\mathbf{x}^{\mathrm{T}}(\omega_m) = \left[S(\omega_m, \beta_1) \quad \ldots \quad S(\omega_m, \beta_K) \right] \tag{7}$$

$$\mathbf{b}(\omega_m) = \mathbf{A}_m\big(\boldsymbol{RAO}\mathrm{s}(\omega_m)\big) \cdot \mathbf{x}(\omega_m) \tag{8}$$

$$\mathbf{b} = \mathbf{A}\mathbf{x} = \begin{bmatrix} \mathbf{b}(\omega_1) \\ \vdots \\ \mathbf{b}(\omega_M) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & & 0 \\ & \ddots & \\ 0 & & \mathbf{A}_M \end{bmatrix} \begin{bmatrix} \mathbf{x}(\omega_1) \\ \vdots \\ \mathbf{x}(\omega_M) \end{bmatrix} \tag{9}$$

The aforementioned linear system defines the first part of the Bayesian estimation algorithm, or the likelihood, but it is easy to check that this is an ill-posed problem, i.e., there are more variables than equations relating it. It means that the uniqueness of the solution can only be achieved if a proper prior probability is defined.

This work follows the propositions of Refs. [9] and [11]. Essentially, they propose three different priors: the first one guarantees the smoothness of the directional spectrum function with the variation of the direction, Eq. (10); the second one guarantees the smoothness of the directional spectrum function with the variation of the angular frequency, Eq. (11); and the third one penalizes energy in regions that are not correctly predicted by the vessel dynamics, with extremely high or low angular frequencies, identified by the indices iLow and iHigh, Eq. (13). The smoothness properties are measured by the summation of the second partial differences between neighbor values, ε, expressed in matrix form using equivalent matrices $\mathbf{H}$, Eq. (12). A unique hyperparameter u is stablished for each particular prior, reaching the final functional $J(\mathbf{x})$ to be minimized in Eq. (14)

$$\varepsilon_{1mk} = S(\omega_m, \beta_{k-1}) - 2S(\omega_m, \beta_k) + S(\omega_m, \beta_{k+1}) \tag{10}$$

$$\varepsilon_{2mk} = S(\omega_{m-1}, \beta_k) - 2S(\omega_m, \beta_k) + S(\omega_{m+1}, \beta_k) \tag{11}$$

$$\sum \varepsilon_{1mk}^2 = \mathbf{x}^{\mathrm{T}}\mathbf{H}_1\mathbf{x} \sum \varepsilon_{2mk}^2 = \mathbf{x}^T\mathbf{H}_2\mathbf{x} \tag{12}$$

$$\sum_{m=1}^{\mathrm{iLow}} \sum_k S(\omega_m, \beta_k)^2 + \sum_{m=\mathrm{iHigh}}^{M} \sum_k S(\omega_m, \beta_k)^2 = \mathbf{x}^{\mathrm{T}}\mathbf{H}_3\mathbf{x} \tag{13}$$

$$J(\mathbf{x}) = \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2 + \mathbf{x}^{\mathrm{T}}[u_1^2\mathbf{H}_1 + u_2^2\mathbf{H}_2 + u_3^2\mathbf{H}_3]\mathbf{x} \tag{14}$$

Following the suggestion from Ref. [4], the vessel motions that are used in this work are heave, sway, and pitch; and the hyperparameters are defined following the procedure described by Bispo et al. [16] for this set of motions.

Assuming that the wave-elevation pattern, measured by the wave-probes, is also linear, it can also be defined using a proper RAO and incorporated in the linear system as a new DOF of the vessel. The wave-probe response, however, suffers from interference from the vessel motions, and does not measure exactly the wave-elevation. It means that an extended linear model must be used.

The Extended Linear Model

Assuming that the rotational motions of the vessel hardly ever surpass 0.1 radians/wave-amplitude (m) and that the vessel only has significant motions for large wave-heights, i.e., the wave-elevation varies slowly along the water surface, only the linearized influence of the degrees-of-freedom that induce vertical motions—heave, roll, and pitch—must be taken in account in order to correct the wave-probe measurements.

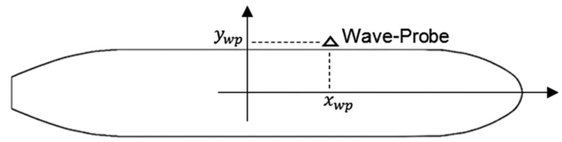

Fig. 4 Position of the wave-probe, following the vessel frame of reference

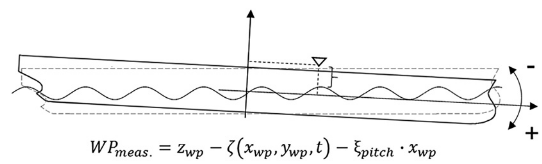

Fig. 5 Linearized influence of the pitch motion in the wave-probe measurement

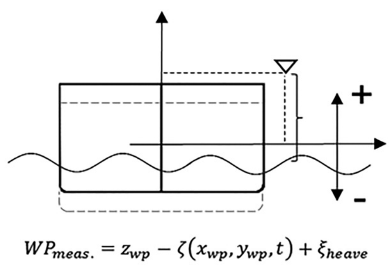

Fig. 6 Influence of the heave motion in the wave-probe measurement

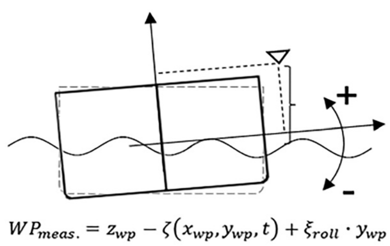

Fig. 7 Linearized influence of the roll motion in the wave-probe measurement

The correction is simply the superposition of the wave-pattern and the wave-probe motions accordingly with its position, which can be stated as a linear transformation following the relations shown in Figs. 4–7.

The final formulation of the superposition is given in the matrix format in Eqs. (15) and (16), with A_{wave} the wave amplitude, $\boldsymbol{\xi}$ the vessel motion, and $\boldsymbol{\zeta}$ the wave elevation.

$$\begin{bmatrix} \xi_{\mathrm{vessel}} \\ \xi_{\mathrm{probe}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{X} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \xi_{\mathrm{vessel}} \\ \zeta_{\mathrm{elevation}} \end{bmatrix} = A_{\mathrm{wave}} \begin{bmatrix} \mathbf{RAO}_{\mathrm{vessel}} \\ \mathbf{RAO}_{\mathrm{probe}} \end{bmatrix} \tag{15}$$

$$\mathbf{X}_i = \left[0 \quad 0 \quad -1 \quad -y_{wp} \quad +x_{wp} \quad 0 \right] \tag{16}$$

Description of the Experimental Campaign

In order to validate the idea, an experimental campaign was performed in a wave-basin, with a small-scale very large crude carrier model with a scale 1:90.

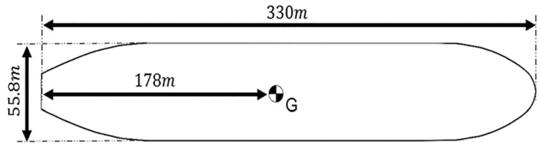

Fig. 8 Main dimensions of the real vessel

Table 1 Inertial properties of the real vessel

Draft (m)	Zcg (m)	Rxx/B	Ryy/L	Rzz/L	Disp (ton)
15	13.9	39%	25%	24%	220,403

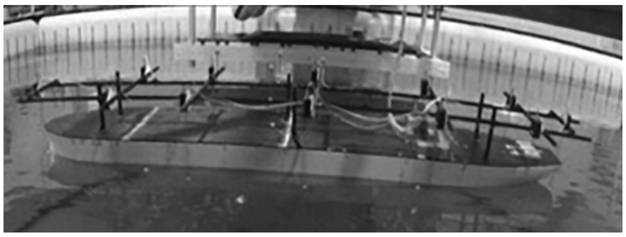

Fig. 9 Setup for the experimental campaign

The main dimensions of the real vessel are shown in Fig. 8, and the inertial properties are shown in Table 1.

The desired properties of the vessel were experimentally validated using static inclination tests, to assure the center of gravity, and decay tests, to assure the resonance period, i.e., the inertial properties. The resonance periods of the model were all within a one-second tolerance regarding the periods of the theoretical RAOs calculated numerically by potential theory.

The wave basin that was used is the hydrodynamic calibrator at numerical towing tank (CH-TPN). Its dimensions are 14 m × 14 m × 4.1 m depth, being a wave basin dedicated to small-scale experiments. It is capable of generating waves along all its squared perimeter, using 148 moving wave generators, or flaps, and has active wave absorption.

The vessel motions were measured by an optical tracking system, which monitors reflective dummies attached on the hull, a nonintrusive method with linear displacement uncertainty around 0.1 mm.

The wave-elevation was measured by 12 wave-probes positioned around the vessel. The employed sensors are capacitive sensors with uncertainty around 1 mm.

The final setup can be seen in Fig. 9.

The directional wave-spectra were generated following the classical Joint North Sea Wave Project parametric spectral energy density, [23], Eqs. (18)–(20), with unidirectional spread function, Eq. (17)

$$S(\omega, \beta) = D(\beta) \cdot S(\omega) = \delta(\beta - \beta_0) \cdot S(\omega) \tag{17}$$

$$S(\omega) = \frac{320 H_S^2}{T_P^4} \omega^{-5} \exp\left\{ \frac{-1950}{T_P^4} \omega^{-4} \right\} \gamma^A \tag{18}$$

$$\gamma = 3.3, A(\omega) = \exp\left\{ -\left(\frac{\omega/\omega_P - 1}{\sigma\sqrt{2}} \right)^2 \right\} \tag{19}$$

$$\sigma = 0.07 \text{ if } \omega < \omega_P, \sigma = 0.09 \text{ otherwise} \tag{20}$$

The parameters H_S and T_P are called, respectively, significant height and peak period. A total of 29 combinations of those two parameters was selected based on statistical data from the metocean of Bacia de Campos—Campos Basin—originated from a region located on the southeast Brazilian coast in the state of Rio de Janeiro. Furthermore, for each combination, a set of five possible incoming directions was used: 0 deg, 180 deg, 225 deg, 270 deg, and 315 deg. The selected parameters are shown in Table 2.

Finally, five possible wave-probe configurations were considered, using from only vessel DOFs (conf. I) to vessel DOFs plus four wave-probes (conf. V), Table 3, and the possible positions were defined using a cross pattern, Fig. 10.

Table 2 Parameters used during the experiments

T_p	5.5	6.4	7.1	7.2	8.1	8.1	8.1
H_s	1.2	1.9	0.9	2.4	1.3	1.7	3.0
8.5	9.0	9.9	9.9	9.9	9.9	10.1	10.9
2.2	3.0	2.0	2.8	3.3	3.8	1.0	3.5
T_p	11.3	11.3	11.3	11.8	11.8	12.8	13.2
H_s	1.8	2.3	2.8	1.3	3.5	4.3	2.3
13.2	13.2	13.7	13.7	14	14.2	14.7	
3.3	3.8	1.8	2.8	0.8	4.3	1.3	

Table 3 Wave-probe configurations

Configuration	Degrees-of-freedom
I	Sway-heave-pitch
II	Sway-heave-pitch WP4
III	Sway-heave-pitch WP4-2
IV	Sway-heave-pitch WP4-2-1
V	Sway-heave-pitch WP4-2-1-3

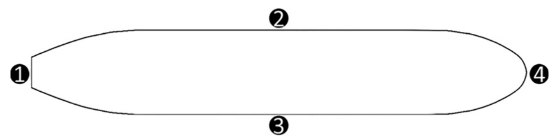

Fig. 10 Possible wave-probe positions

Results and Discussion

This section presents a summary of all the results obtained, aiming to provide answers for the following points:

- Is the extended linear model, used in the Bayesian estimation, valid for real systems?
- Is the addition of the wave-probes capable of improving the estimation of the energy in low-periods?
- How many wave-probes are enough?
- What are the possible drawbacks of using wave probes?

The first point, the validity of the extended linear model, proposed previously, is easily verified by the spectral estimation of the transfer function of each generalized DOF, not only the vessel motions but also the wave-probe responses. This was done by the nonparametric system identification spectral method described in Ref. [24], for each sea state, using the ratio of the cross-spectra—between the wave signal and the DOF signal—and the spectral energy density. All the spectral quantities were estimated using the Welch method with Hamming windowing, as suggested by the authors. The results were superimposed on the RAOs calculated using potential theory by the software WAMIT [25], in order to compare both, Figs. 11–16.

The resonance period for the roll motion is around 14 s and for the heave motion is around 12 s, both calculated for beam seas.

The analysis of the graphs suggests that the extended linear model is indeed a reasonable approximation for most of the

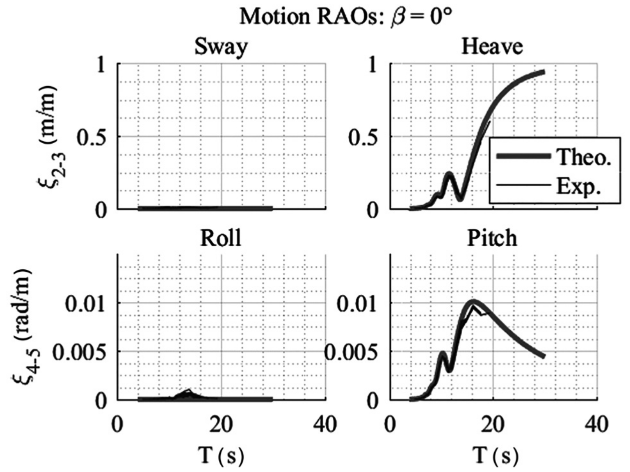

Fig. 11 RAOs from selected vessel motions at 0 deg, which are similar to the ones at 180 deg

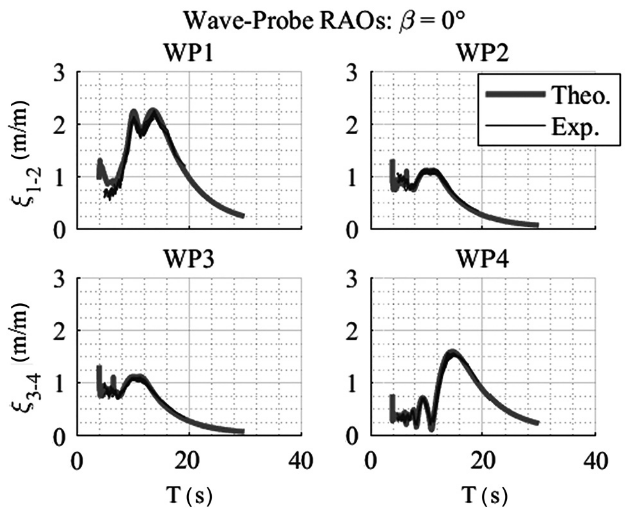

Fig. 12 RAOs from the wave-probes at 0 deg, which are similar to the ones at 180 deg

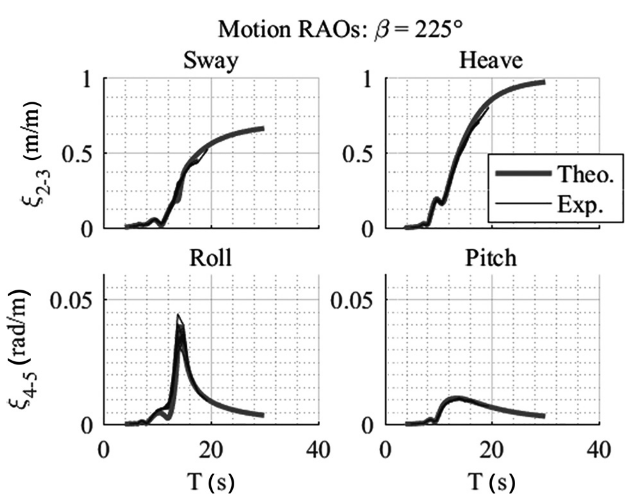

Fig. 13 RAOs from selected vessel motions at 225 deg, which are similar to the ones at 315 deg

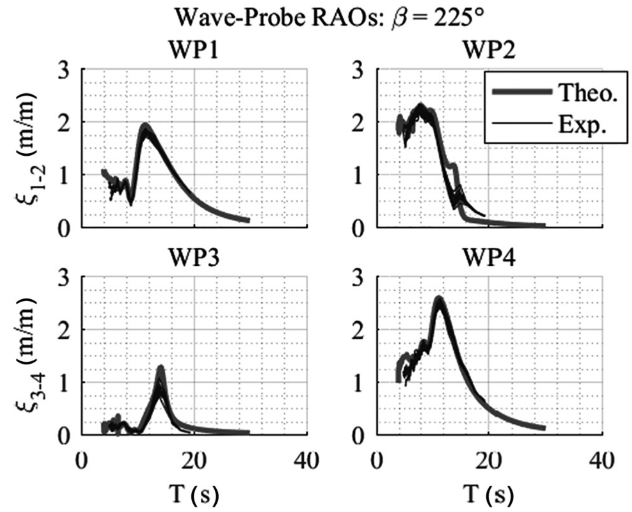

Fig. 14 RAOs from the wave-probes at 225 deg, which are similar to the ones at 315 deg

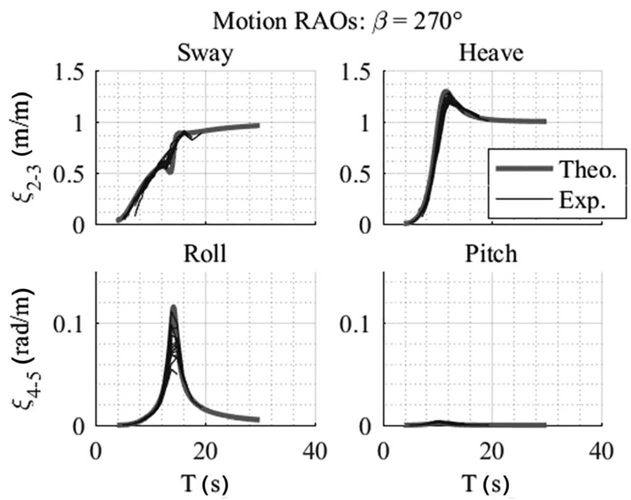

Fig. 15 RAOs from selected vessel motions at 270 deg

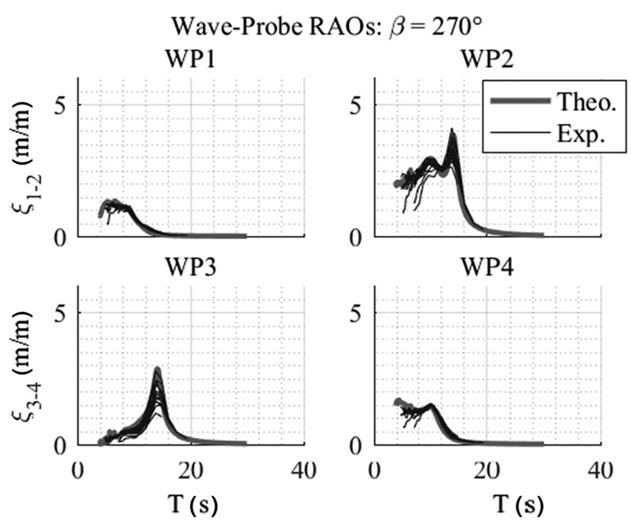

Fig. 16 RAOs from the wave-probes at 270 deg

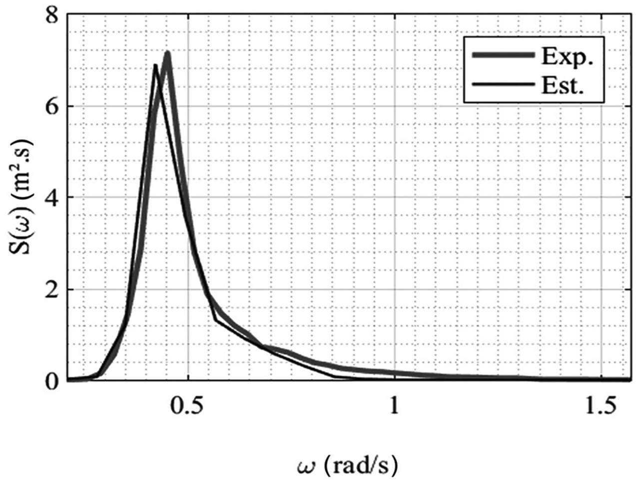

Fig. 17 Experimental and estimated spectral energy, conf. I: $H_s = 4.3$ m, $T_p = 14.0$ s, $\beta_0 = 270$ deg

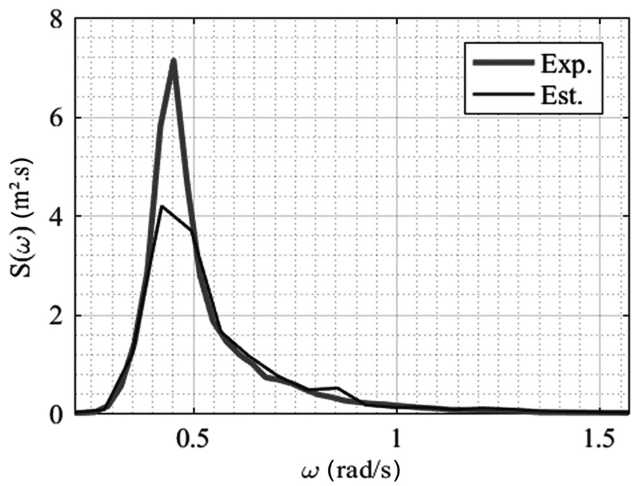

Fig. 18 Experimental and estimated spectral energy, conf. V: $H_s = 4.3$ m, $T_p = 14.0$ s, $\beta_0 = 270$ deg

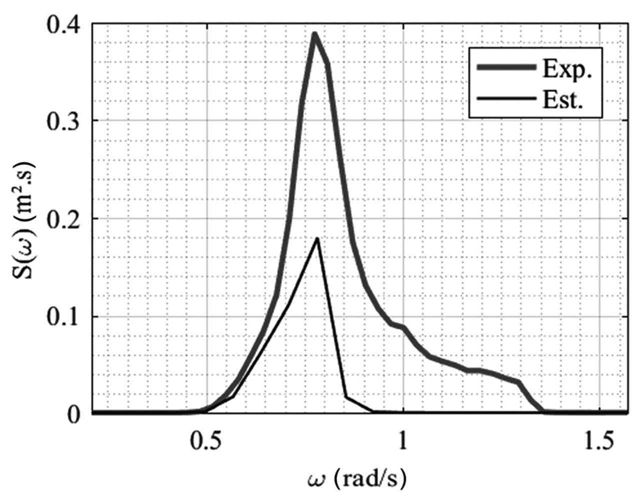

Fig. 19 Experimental and estimated spectral energy, conf. I: $H_s = 1.3$ m, $T_p = 8.0$ s, $\beta_0 = 0$ deg

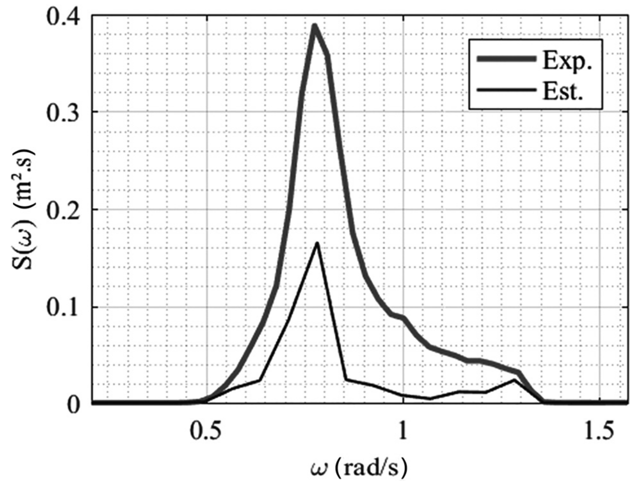

Fig. 20 Experimental and estimated spectral energy, conf. II: $H_s = 1.3$ m, $T_p = 8.0$ s, $\beta_0 = 0$ deg

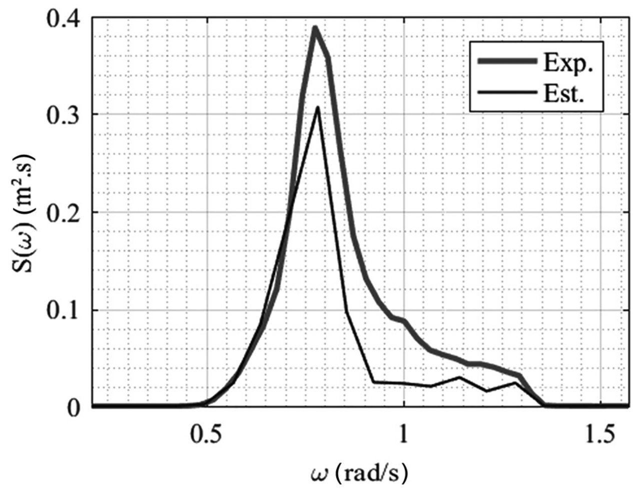

Fig. 21 Experimental and estimated spectral energy, conf. III: $H_s = 1.3$ m, $T_p = 8.0$ s, $\beta_0 = 0$ deg

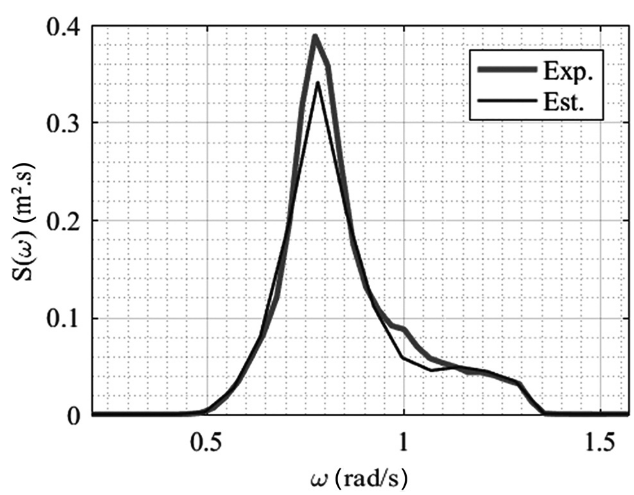

Fig. 22 Experimental and estimated spectral energy, conf. IV: $H_s = 1.3$ m, $T_p = 8.0$ s, $\beta_0 = 0$ deg

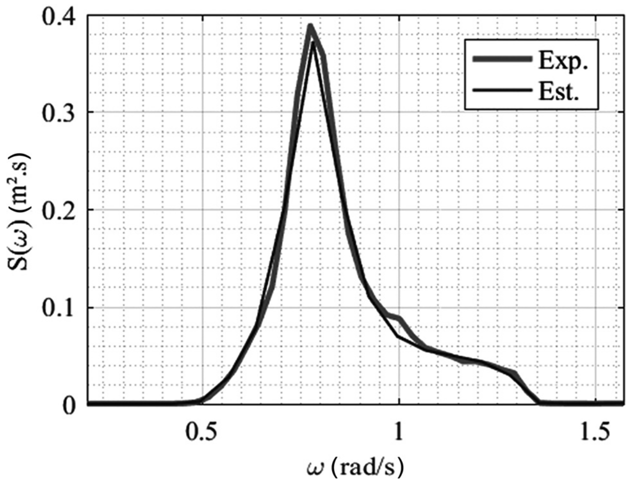

Fig. 23 Experimental and estimated spectral energy, conf. V: $H_s = 1.3$ m, $T_p = 8.0$ s, $\beta_0 = 0$ deg

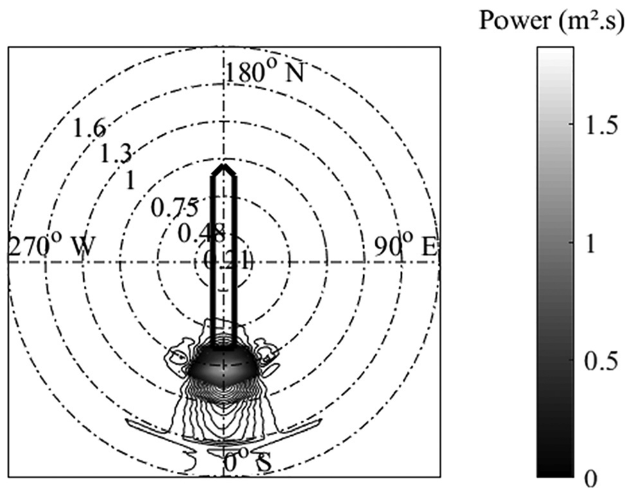

Fig. 24 Experimental directional spectrum: exp.: $H_s = 1.3$ m, $T_p = 8.0$ s, $\beta_0 = 0$ deg

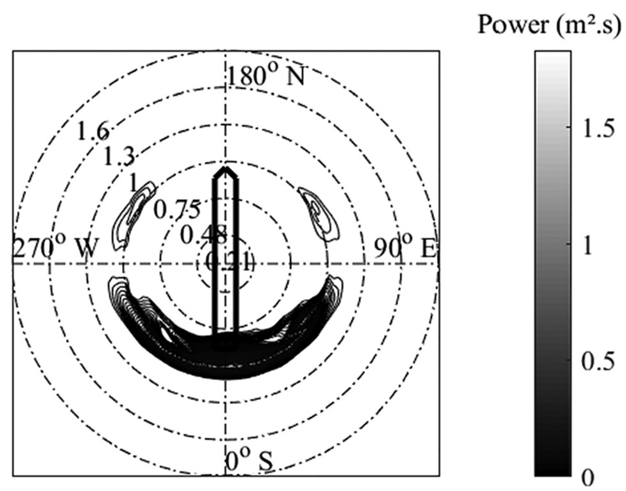

Fig. 25 Estimated directional spectrum using configuration I: $H_s = 1.3$ m, $T_p = 8.0$ s, $\beta_0 = 0$ deg

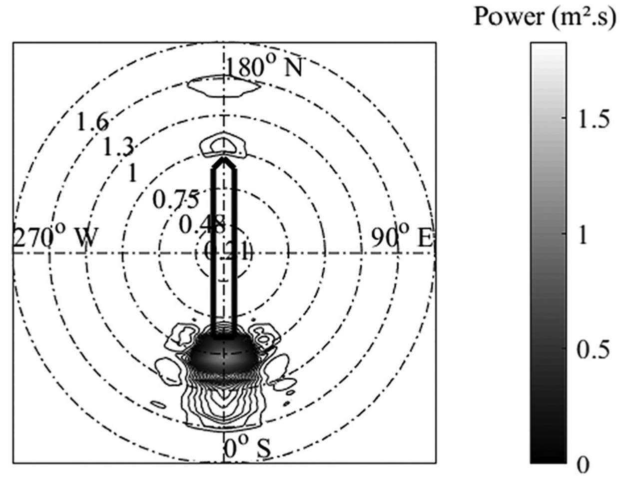

Fig. 26 Estimated directional spectrum using configuration V: $H_s = 1.3$ m, $T_p = 8.0$ s, $\beta_0 = 0$ deg

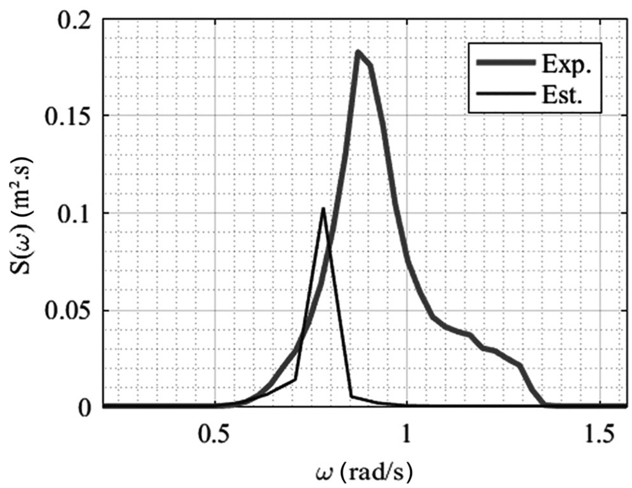

Fig. 27 Experimental and estimated spectral energy, conf. I: $H_s = 0.9$ m, $T_p = 7.0$ s, $\beta_0 = 225$ deg

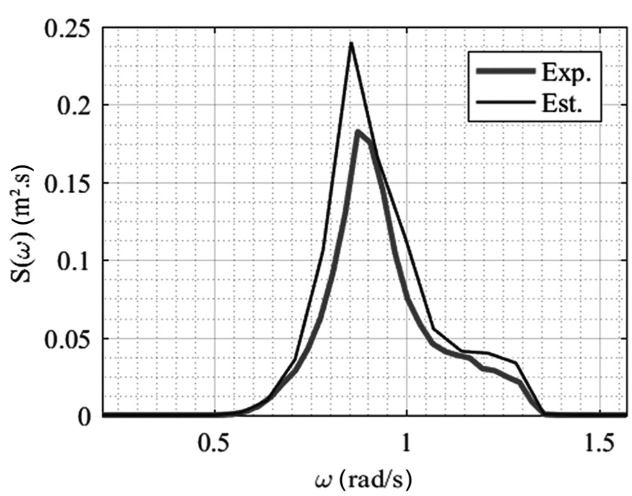

Fig. 28 Experimental and estimated spectral energy, conf. V: $H_s = 0.9$ m, $T_p = 7.0$ s, $\beta_0 = 225$ deg

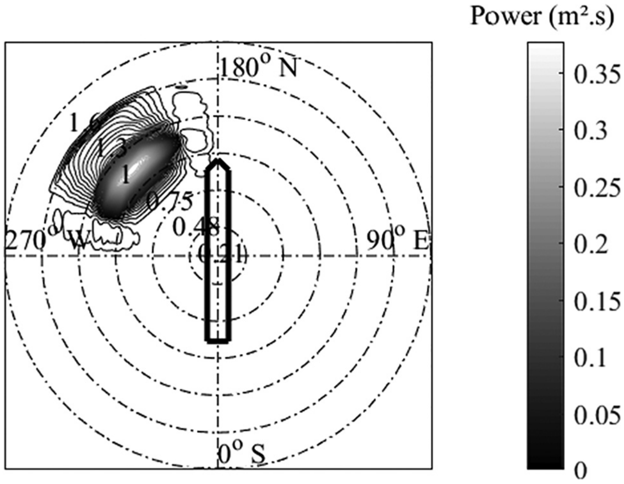

Fig. 29 Experimental directional spectrum: exp.: $H_s = 0.9$ m, $T_p = 7.0$ s, $\beta_0 = 225$ deg

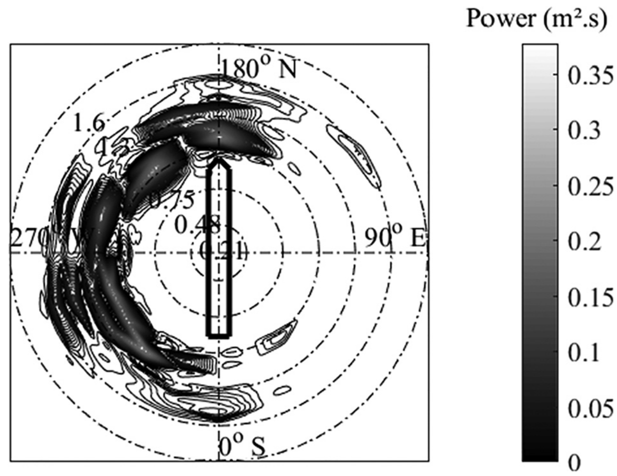

Fig. 31 Estimated directional spectrum using configuration V: $H_s = 0.9$ m, $T_p = 7.0$ s, $\beta_0 = 225$ deg

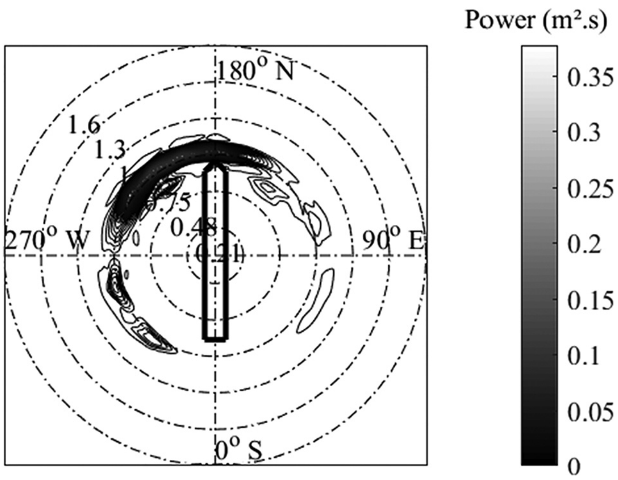

Fig. 30 Estimated directional spectrum using configuration I: $H_s = 0.9$ m, $T_p = 7.0$ s, $\beta_0 = 225$ deg

DOFs. It is important to notice, however, that the roll motion, mainly at 270 deg, has non-negligible nonlinear effects, as discussed by Tannuri et al. [4]. A more detailed analysis reveals that the higher the roll motion, the less is the transfer function, suggesting a nonlinear damping effect.

The roll nonlinearity causes the first drawback of incorporating wave-probes. Since the extended linear model incorporates the vessel motions to calculate the gauge response, the nonlinearity from roll has influence in all the probes, except the ones in the bow and the stern, which can degrade the estimation capabilities of the method close to the roll resonance, mainly for high wave amplitudes.

An example of the discussed problem is given in Figs. 17 and 18. The figures present the estimated spectral density energy for two configurations, the first one, without roll effects, and the last one, with two probes with roll influence. Since the HS value is high, the nonlinearity in roll is also high and the motion will be smaller than the predicted by the linear theory, resulting in a subestimation.

Despite the previous case, with a high HS, the results generally support the wave-probe approach, Figs. 19–23.

The complete directional spectrum estimation can be seen in Figs. 24–26.

The previous result, that represents the general behavior of the method, drives to some important conclusions. First, the addition of the wave-probes is indeed able to improve the estimation capabilities of energy coming from periods lower than the cutoff period, i.e., frequencies greater than 0.8 rad/s. Second, the addition of the wave-probes was also able to improve the estimation capabilities greater than the cutoff period, i.e., it can improve the estimation capabilities in the entire range of the spectrum. Last, a convergence appears to happen with the addition of four wave-probes, configuration V, providing evidence for the claim that this method is able to use a reduced number of wave-probes.

The conclusions are valid for all the tested conditions, mainly in the directions 0 deg, 180 deg, and 270 deg; excluding the case at 270 deg, close to the roll resonance and with H_S greater than 3 m.

At 225 deg, however, another drawback happens for low peak periods. Despite the improvement in the energy estimation, Figs. 27 and 28, the directional spectrum estimation has a considerable error, Figs. 29–31.

This kind of error probably happens because for low T_P the wave-probe response is much higher than the vessel motions response, in a way that the estimation algorithm acts as if only the probe response was being taken in account. The problem of this is that a single wave-probe at the hull of a nonmoving vessel is not able to discriminate the incoming direction of the waves, only the energy, degrading the estimation capabilities of the algorithm.

Possible solutions for this problem may involve a more robust method for weighting the responses of each generalized DOF, increasing the importance of the vessel motions for this particular set of cases.

Conclusions

Restating the main objective of studying the incorporation of wave-probe measurements in the motion-based wave estimation, in order to avoid the filtering problem described in the literature, this work proposed a simple solution based on Bayesian estimation algorithms.

An extended linear model approach was proposed and validated by experimental tests in a wave-basin. The experiments were also used to validate the wave-probe incorporation approach, concluding that the addition of the probes is able to improve not only the estimation capabilities of energies coming from periods lower than the cutoff period but also the estimation capabilities in the entire range of the spectrum.

Some drawbacks were discussed: the nonlinearity effects from roll, which affects the wave-probes, due to the extended linear model approach; and the bad directional predictions is some particular directions, when low T_P conditions happen and only the wave-probe measurements are taken in account.

Further studies are necessary to overcome those problems, but it is already possible to conclude that the method has, indeed, potential to extent the Bayesian estimation for dynamic positioning applications, which demands good estimation capabilities for low-period waves, without the need of a large number of sensors, as it is usually required by the standard practice in the area.

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